SAMPLE PAPER

issued by CBSE for Board Exams (2022-23)
Mathematics (041) - Class 12

Time Allowed : 180 Minutes | **Max. Marks : 80**

General Instructions :

1. This Question paper contains **five sections - A, B, C, D and E**. Each section is compulsory. However, there are **internal choices** in some questions.
2. Section A has **18 MCQs** and **02 Assertion-Reason (A-R)** based questions of **1 mark** each.
   Section B has **05 questions** of **2 marks** each.
   Section C has **06 questions** of **3 marks** each.
   Section D has **04 questions** of **5 marks** each.
   Section E has **03 Case-study / Source-based / Passage-based** questions with **sub-parts** (4 **marks** each).
3. There is no overall choice. However, **internal choice** has been provided in
   - **02 Questions of Section B**
   - **03 Questions of Section C**
   - **02 Questions of Section D**
   - **02 Questions of Section E**

   You have to attempt only one of the alternatives in all such questions.

SECTION A

*(Question numbers 01 to 20 carry **1 mark** each.)*
*Followings are **multiple choice questions**. Select the correct option in each one of them.*

01. If $A=\left[a_{ij}\right]$ is a skew-symmetric matrix of order n, then

(a) $a_{ij}=\frac{1}{a_{ji}}\ \forall\ i, j$ (b) $a_{ij}\neq 0\ \forall\ i, j$

(c) $a_{ij}=0$, where $i=j$ (d) $a_{ij}\neq 0$ where $i=j$

02. If A is a square matrix of order 3, $|A'|=-3$, then $|AA'|=$

(a) 9 (b) −9 (c) 3 (d) −3

03. The area of a triangle with vertices A, B, C is given by

(a) $\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|$ (b) $\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|$ (c) $\frac{1}{4}\left|\overrightarrow{AC}\times\overrightarrow{AB}\right|$ (d) $\frac{1}{8}\left|\overrightarrow{AC}\times\overrightarrow{AB}\right|$

04. The value of 'k' for which the function $f(x)=\begin{cases}\frac{1-\cos 4x}{8x^2}, & \text{if } x\neq 0\\ k, & \text{if } x=0\end{cases}$ is continuous at $x=0$, is

(a) 0 (b) −1 (c) 1 (d) 2

05. If $f'(x)=x+\frac{1}{x}$, then $f(x)$ is

(a) $x^2+\log|x|+C$ (b) $\frac{x^2}{2}+\log|x|+C$ (c) $\frac{x}{2}+\log|x|+C$ (d) $\frac{x}{2}-\log|x|+C$

06. If m and n respectively, are the order and the degree of the differential equation $\frac{d}{dx}\left[\left(\frac{dy}{dx}\right)\right]^4 = 0$, then $m+n=$

(a) 1 (b) 2 (c) 3 (d) 4

07. The solution set of the inequality $3x+5y<4$ is

(a) an open half-plane not containing the origin
(b) an open half-plane containing the origin
(c) the whole XY-plane not containing the line $3x+5y=4$
(d) a closed half-plane containing the origin

08. The scalar projection of the vector $3\hat{i}-\hat{j}-2\hat{k}$ on the vector $\hat{i}+2\hat{j}-3\hat{k}$ is

(a) $\frac{7}{\sqrt{14}}$ (b) $\frac{7}{14}$ (c) $\frac{6}{13}$ (d) $\frac{7}{2}$

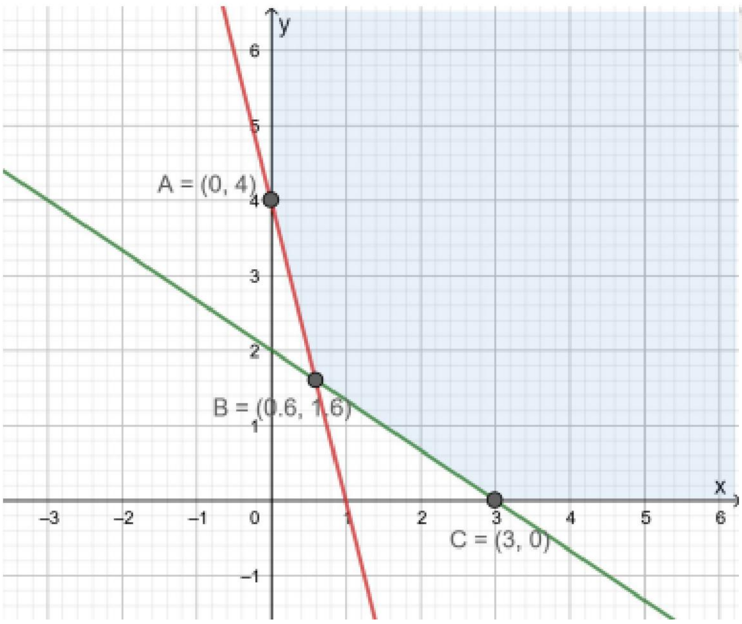
09. The value of $\int_2^3 \frac{x}{x^2+1}dx$ is

(a) $\log 4$ (b) $\log\frac{3}{2}$ (c) $\frac{1}{2}\log 2$ (d) $\log\frac{9}{4}$

10. If A, B are non-singular square matrices of the same order, then $(AB^{-1})^{-1} =$

(a) $A^{-1}B$ (b) $A^{-1}B^{-1}$ (c) BA^{-1} (d) AB

11. The corner points of the shaded unbounded feasible region of an LPP are (0, 4), (0.6, 1.6) and (3, 0) as shown in the figure.
The minimum value of the objective function $Z=4x+6y$ occurs at



(a) (0.6, 1.6) only
(b) (3, 0) only
(c) (0.6, 1.6) and (3, 0) only
(d) every point of the line-segment joining the points (0.6, 1.6) and (3, 0)

12. If $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$, then the possible value(s) of 'x' is/are

(a) 3 (b) $\sqrt{3}$ (c) $-\sqrt{3}$ (d) $\sqrt{3}, -\sqrt{3}$

13. If A is a square matrix of order 3 and $|A|=5$, then $|\text{adj} A| =$

(a) 5 (b) 25 (c) 125 (d) $\frac{1}{5}$

14. Given two independent events A and B such that $P(A)=0.3$, $P(B)=0.6$ and $P(A'\cap B')$ is

(a) 0.9 (b) 0.18 (c) 0.28 (d) 0.1

15. The general solution of the differential equation $ydx-xdy=0$ is

(a) $xy=C$ (b) $x=Cy^2$ (c) $y=Cx$ (d) $y=Cx^2$

16. If $y=\sin^{-1}x$, then $(1-x^2)y_2$ is equal to

(a) xy_1 (b) xy (c) xy_2 (d) x^2

17. If two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}|=2$, $|\vec{b}|=3$ and $\vec{a}.\vec{b}=4$, then $|\vec{a}-2\vec{b}|$ is equal to

(a) $\sqrt{2}$ (b) $2\sqrt{6}$ (c) 24 (d) $2\sqrt{2}$

18. P is a point on the line joining the points A(0, 5, –2) and B(3, –1, 2). If the x-coordinate of P is 6, then its z-coordinate is

(a) 10 (b) 6 (c) −6 (d) −10

*Followings are **Assertion-Reason based questions**.*

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true and R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19. **Assertion (A) :** The domain of the function $\sec^{-1}2x$ is $\left(-\infty,-\frac{1}{2}\right]\cup\left[\frac{1}{2},\infty\right)$.

Reason (R) : $\sec^{-1}(-2)=-\frac{\pi}{4}$.

20. **Assertion (A) :** The acute angle between the line $\vec{r}=\hat{i}+\hat{j}+2\hat{k}+\lambda(\hat{i}-\hat{j})$ and the x-axis is $\frac{\pi}{4}$.

Reason (R) : The acute angle θ between the lines

$\vec{r}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}+\lambda(a_1\hat{i}+b_1\hat{j}+c_1\hat{k})$ and

$\vec{r}=x_2\hat{i}+y_2\hat{j}+z_2\hat{k}+\mu(a_2\hat{i}+b_2\hat{j}+c_2\hat{k})$ is given by $\cos\theta=\dfrac{|a_1a_2+b_1b_2+c_1c_2|}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$.

SECTION B

*(Question numbers 21 to 25 carry **2 marks** each.)*

21. Find the value of $\sin^{-1}\left[\sin\left(\frac{13\pi}{7}\right)\right]$.

OR

Prove that the function f is surjective, where $f:N\to N$ such that

$$f(n)=\begin{cases}\frac{n+1}{2}, & \text{if } n \text{ is odd}\\ \frac{n}{2}, & \text{if } n \text{ is even}\end{cases}$$

Is the function injective? Justify your answer.

22. A man 1.6 m tall walks at the rate of 0.3 m/sec away from a street light that is 4 m above the ground. At what rate is the tip of his shadow moving? At what rate is his shadow lengthening?

23. If $\vec{a}=\hat{i}-\hat{j}+7\hat{k}$ and $\vec{b}=5\hat{i}-\hat{j}+\lambda\hat{k}$, then find the value of λ so that the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are orthogonal.

OR

Find the direction ratios and direction cosines of a line parallel to the line whose equation is $6x-12=3y+9=2z-2$.

24. If $y\sqrt{1-x^2}+x\sqrt{1-y^2}=1$, then prove that $\dfrac{dy}{dx}=-\sqrt{\dfrac{1-y^2}{1-x^2}}$.

25. Find $|\vec{x}|$, if $(\vec{x}-\vec{a}).(\vec{x}+\vec{a})=12$, where $\vec{a}$ is a unit vector.

SECTION C

*(Question numbers 26 to 31 carry **3 marks** each.)*

26. Find : $\displaystyle\int\frac{dx}{\sqrt{3-2x-x^2}}$.

27. Three friends go for coffee. They decide who will pay the bill, by each tossing a coin and then letting the "odd person" pay. There is no odd person if all three tosses produce the same result. If there is no odd person in the first round, they make a second round of tosses and they continue to do so until there is an odd person. What is the probability that exactly three rounds of tosses are made?

OR

Find the mean number of defective items in a sample of two items drawn one-by-one without replacement from an urn containing 6 items, which include 2 defective items. Assume that the items are identical in shape and size.

28. Evaluate : $\displaystyle\int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\tan x}}$.

OR

Evaluate : $\displaystyle\int_0^4|x-1|\,dx$.

29. Solve the differential equation : $ydx+(x-y^2)dy=0$.

OR

Solve the differential equation : $xdy-ydx=\sqrt{x^2+y^2}\,dx$.

30. Solve the following Linear Programming Problem graphically :
Maximize $Z=400x+300y$ subject to $x+y\le 200$, $x\le 40$, $x\ge 20$, $y\ge 0$.

31. Find : $\displaystyle\int\frac{(x^3+x+1)}{(x^2-1)}dx$.

SECTION D

*(Question numbers 32 to 35 carry **5 marks** each.)*

32. Make a rough sketch of the region $\{(x,y):0\le y\le x^2,\ 0\le y\le x,\ 0\le x\le 2\}$ and find the area of the region using integration.

33. Define the relation R in the set $N\times N$ as follows :
For $(a,b),(c,d)\in N\times N$, $(a,b)\ R\ (c,d)$ iff $ad=bc$.
Prove that R is an equivalence relation in $N\times N$.

OR

Given a non-empty set X, define the relation R in $P(X)$ as follows :
For $A, B \in P(X), (A, B) \in R$ iff $A \subset B$.
Prove that R is reflexive, transitive and not symmetric.

34. An insect is crawling along the line $\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$ and another insect is crawling along the line $\vec{r} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$. At what points on the lines should they reach so that the distance between them is the shortest? Find the shortest possible distance between them.

OR

The equations of motion of a rocket are :
$x = 2t,\ y = -4t,\ z = 4t$, where the time t is given in seconds, and the coordinates of a moving point in km. What is the path of the rocket? At what distances will the rocket be from the starting point $O(0, 0, 0)$ and from the following line in 10 seconds?
$\vec{r} = 20\hat{i} - 10\hat{j} + 40\hat{k} + \mu(10\hat{i} - 20\hat{j} + 10\hat{k})$.

35. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Use A^{-1} to solve the following system of equations
$2x - 3y + 5z = 11,\ 3x + 2y - 4z = -5,\ x + y - 2z = -3$.

SECTION E

*(Question numbers 36 to 38 carry **4 marks** each.)*
*This section contains **three Case-study / Passage based questions**.*
*First two questions have **three sub-parts** (i), (ii) and (iii) of **marks 1, 1 and 2** respectively.*
*Third question has **two sub-parts** of **2 marks** each.*

36. **CASE STUDY I :** Read the following passage and the answer the questions given below.

The temperature of a person during an intestinal illness is given by
$f(x) = -0.1x^2 + mx + 98.6,\ 0 \le x \le 12$, m being a constant, where $f(x)$ is the temperature in °F at x days.

(i) Is the function differentiable in the interval $(0,12)$? Justify your answer.
(ii) If 6 is the critical point of the function, then find the value of the constant m.
(iii) Find the intervals in which the function is strictly increasing / strictly decreasing.

OR

(iii) Find the points of local maximum / local minimum, if any, in the interval (0, 12) as well as the points of absolute maximum / absolute minimum in the interval $[0, 12]$. Also, find the corresponding local maximum / local minimum and the absolute maximum / absolute minimum values of the function.

37. **CASE STUDY II :** Read the following passage and answer the questions given below.

In an elliptical sport field the authority wants to design a rectangular soccer field with the maximum possible area. The sport field is given by the graph of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

(i) If the length and the breadth of the rectangular field be $2x$ and $2y$ respectively, then find the area function in terms of x.

(ii) Find the critical point of the function.

(iii) Use First Derivative test to find the length $2x$ and width $2y$ of the soccer field (in terms of a and b) that maximize its area.

OR

(iii) Use Second Derivative test to find the length $2x$ and width $2y$ of the soccer field (in terms of a and b) that maximize its area.

38. **CASE STUDY III :** Read the following passage and answer the questions given below.

There are two anti craft guns, named as A and B. The probabilities that the shell fired from them hits an airplane are 0.3 and 0.2 respectively. Both of them fired one shell at an airplane at the same time.

(i) What is the probability that the shell fired from exactly one of them hit the plane?

(ii) If it is known that the shell fired from exactly one of them hit the plane, then what is the probability that it was fired from B?

DETAILED SOLUTIONS

SECTION A

01. (c) In a skew-symmetric matrix, $a_{ij} = -a_{ji}$. Hence for $i = j$, we have $a_{ii} = 0$.

02. (a) $|AA'| = |A||A'| = |A||A| = (-3)(-3) = 9$.

03. (b) The area of the parallelogram with adjacent sides AB and AC $= |\overrightarrow{AB} \times \overrightarrow{AC}|$.

Hence, the area of the triangle with vertices A, B, C $= \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$.

04. (c) The function f is continuous at $x = 0$, if $\lim_{x\to 0} f(x) = f(0)$ …(i)

We have $f(0) = k$ and $\lim_{x\to 0} f(x) = \lim_{x\to 0}\frac{1-\cos 4x}{8x^2} = \lim_{x\to 0}\frac{2\sin^2 2x}{8x^2} = \lim_{x\to 0}\frac{\sin^2 2x}{4x^2}$

$\Rightarrow \lim_{x\to 0}\left(\frac{\sin 2x}{2x}\right)^2 = 1$

Hence by (i), $k = 1$.

05. (b) $\because f(x) = \int f'(x)dx$ $\quad\therefore f(x) = \int\left(x + \frac{1}{x}\right)dx$

So, $f(x) = \frac{x^2}{2} + \log|x| + C$.

06. (c) The given differential equation is $4\left(\frac{dy}{dx}\right)^3\left(\frac{d^2y}{dx^2}\right) = 0$. Here, $m = 2$ and $n = 1$.

Hence, $m + n = 3$.

07. (b) The strict inequality represents an open half plane and it contains the origin as (0, 0) satisfies the inequality $3x + 5y < 4$.

08. (a) Scalar Projection of $3\hat{i} - \hat{j} - 2\hat{k}$ on vector $\hat{i} + 2\hat{j} - 3\hat{k} = \frac{(3\hat{i} - \hat{j} - 2\hat{k}).(\hat{i} + 2\hat{k} - 3\hat{k})}{|\hat{i} + 2\hat{j} - 3\hat{k}|} = \frac{7}{\sqrt{14}}$.

09. (c) $\int_2^3 \frac{x}{x^2+1}dx = \frac{1}{2}\int_2^3 \frac{2x}{x^2+1}dx = \frac{1}{2}\left[\log(x^2+1)\right]_2^3 = \frac{1}{2}(\log 10 - \log 5) = \frac{1}{2}\log\left(\frac{10}{5}\right) = \frac{1}{2}\log 2$.

10. (c) $(AB^{-1})^{-1} = (B^{-1})^{-1}A^{-1} = BA^{-1}$.

11. (d) The minimum value of the objective function occurs at two adjacent corner points (0.6, 1.6) and (3, 0) and there is no point in the half plane $4x + 6y < 12$ which is common with the feasible region. So, the minimum value occurs at every point of the line segment joining the two points.

12. (d) Note that, $2\times 1 - 5\times 4 = 2x^2 - 24$

$\Rightarrow 2x^2 = 6 \quad \Rightarrow x^2 = 3$

$\Rightarrow x = \pm\sqrt{3}$.

13. (b) $\because |\text{adj} A| = |A|^{n-1} \quad \therefore |\text{adj} A| = |A|^{3-1}$

$\Rightarrow |\text{adj} A| = 25$.

14. (c) $P(A' \cap B') = P(A') \times P(B')$ (As A and B are independent, A' and B' are also independent.)

$= 0.7 \times 0.4 = 0.28$.

15. (c) $ydx - xdy = 0$

$\Rightarrow ydx = xdy$

$\Rightarrow \int \frac{dy}{y} = \int \frac{dx}{x}$

$\Rightarrow \log|y| = \log|x| + \log K,\ K > 0$

$\Rightarrow \log|y| = \log\left[|x|K\right]$

$\Rightarrow |y| = |x|K \quad \Rightarrow y = \pm Kx$

$\therefore y = Cx$, where $C = \pm K$.

16. (a) $y = \sin^{-1} x \qquad \Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

$\Rightarrow \sqrt{1-x^2}.\frac{dy}{dx} = 1$

Again, differentiating both sides w.r. to x, we get $\sqrt{1-x^2}\frac{d^2y}{dx^2} + \frac{dy}{dx}.\left(\frac{-2x}{2\sqrt{1-x^2}}\right) = 0$

On simplifying, we get $(1-x^2)y_2 = xy_1$.

17. (b) $\left|\vec{a} - 2\vec{b}\right|^2 = (\vec{a} - 2\vec{b}).(\vec{a} - 2\vec{b})$

$\Rightarrow \left|\vec{a} - 2\vec{b}\right|^2 = \vec{a}.\vec{a} - 4\,\vec{a}.\vec{b} + 4\vec{b}.\vec{b}$

$\Rightarrow \left|\vec{a} - 2\vec{b}\right|^2 = \left|\vec{a}\right|^2 - 4\vec{a}.\vec{b} + 4\left|\vec{b}\right|^2$

$\Rightarrow \left|\vec{a} - 2\vec{b}\right|^2 = 4 - 16 + 36 = 24$

$\therefore \left|\vec{a} - 2\vec{b}\right| = 2\sqrt{6}$.

18. (b) The line through the points $(0,\ 5,\ -2)$ and $(3,\ -1,\ 2)$ is $\frac{x}{3-0} = \frac{y-5}{-1-5} = \frac{z+2}{2+2}$

i.e., $\frac{x}{3} = \frac{y-5}{-6} = \frac{z+2}{4} = k\,(\text{say})$

Any point on the line is $P\,(3k,\ -6k+5,\ 4k-2)$, where k is an arbitrary scalar.

As x-coordinate of point P is 6 so, $3k = 6 \ \therefore k = 2$.

The z-coordinate of the point P will be $4 \times 2 - 2 = 6$.

19. (c) Recall that, $\sec^{-1} x$ is defined if $x \le -1$ or $x \ge 1$.

Hence, $\sec^{-1} 2x$ will be defined if $x \le -\frac{1}{2}$ or $x \ge \frac{1}{2}$.

So, A is true.

Also note that, $\sec^{-1}(-2) = \frac{2\pi}{3}$.

So, R is false.

20. (a) The equation of the x-axis may be written as $\vec{r} = \mu(\hat{i})$. Hence, the acute angle θ between the given line and the x-axis is given by $\cos\theta = \frac{\left|1\times1 + (-1)\times0 + 0\times0\right|}{\sqrt{1^2 + (-1)^2 + 0^2}\sqrt{1^2 + 0^2 + 0^2}} = \frac{1}{\sqrt{2}}$

$\Rightarrow \theta = \frac{\pi}{4}$.

So, A is true. Similarly R is true. Also R is the correct explanation of A.

SECTION B

21. $\sin^{-1}\left[\sin\left(\frac{13\pi}{7}\right)\right]=\sin^{-1}\left[\sin\left(2\pi-\frac{\pi}{7}\right)\right]=\sin^{-1}\left[-\sin\left(\frac{\pi}{7}\right)\right]=\sin^{-1}\left[\sin\left(-\frac{\pi}{7}\right)\right]=-\frac{\pi}{7}$.

OR

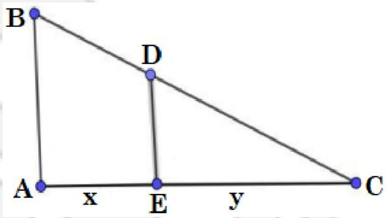
Let $y\in N$ (codomain). Then $\exists\ 2y\in N$ (domain) such that

$f(2y)=\frac{2y}{2}=y$. Hence, f is surjective.

Note that $1, 2\in N$ (domain) such that $f(1)=1=f(2)$.

Hence, f is not injective.

22. Let AB represent the height of the street light from the ground. At any time t seconds, let the man represented as ED of height 1.6 m be at a distance of x m from AB and the length of his shadow EC by y m.



Using similarity of triangles, we've $\frac{4}{1.6}=\frac{x+y}{y}\quad\Rightarrow 3y=2x$

Differentiating both sides w.r. to t, we get $3\times\frac{dy}{dt}=2\times\frac{dx}{dt}$

$\Rightarrow\frac{dy}{dt}=\frac{2}{3}\times 0.3=0.2$.

At any time t seconds, the tip of his shadow is at a distance of $(x+y)$ m from AB.

The rate at which the tip of his shadow moving $=\left(\frac{dx}{dt}+\frac{dy}{dt}\right)$ m/s $=(0.3+0.2)$ m/s $=0.5$ m/s.

Also, the rate at which his shadow is lengthening $=\frac{dy}{dt}$ m/s $=0.2$ m/s.

23. $\vec{a}=\hat{i}-\hat{j}+7\hat{k}$ and $\vec{b}=5\hat{i}-\hat{j}+\lambda\hat{k}$

Hence, $\vec{a}+\vec{b}=6\hat{i}-2\hat{j}+(7+\lambda)\hat{k}$ and $\vec{a}-\vec{b}=-4\hat{i}+(7-\lambda)\hat{k}$

Now $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ will be orthogonal if, $(\vec{a}+\vec{b}).(\vec{a}-\vec{b})=0$

i.e., if, $-24+(49-\lambda^2)=0\quad\Rightarrow\lambda^2=25$

i.e., if, $\lambda=\pm 5$.

OR

Equation of the line $6x-12=3y+9=2z-2$, which when written in standard symmetric form,

will be $\frac{x-2}{\frac{1}{6}}=\frac{y-(-3)}{\frac{1}{3}}=\frac{z-1}{\frac{1}{2}}$

Since, lines are parallel, we have $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

Hence, the required direction ratios are $\frac{1}{6},\frac{1}{3},\frac{1}{2}$ i.e., 1, 2, 3.

And the required direction cosines are $\frac{1}{\sqrt{14}},\frac{2}{\sqrt{14}},\frac{3}{\sqrt{14}}$.

24. Let $\sin^{-1}x=A$ and $\sin^{-1}y=B$. Then $x=\sin A$ and $y=\sin B$

So, $y\sqrt{1-x^2}+x\sqrt{1-y^2}=1$ implies, $\sin B\cos A+\sin A\cos B=1$

$\Rightarrow\sin(A+B)=1$

$\Rightarrow A+B=\sin^{-1}1=\dfrac{\pi}{2}$

$\Rightarrow \sin^{-1}x+\sin^{-1}y=\dfrac{\pi}{2}$

Differentiating w.r. to x, we obtain $\dfrac{1}{\sqrt{1-x^2}}+\dfrac{1}{\sqrt{1-y^2}}\times\dfrac{dy}{dx}=0$

$\Rightarrow \dfrac{dy}{dx}=-\sqrt{\dfrac{1-y^2}{1-x^2}}$.

25. Since $\vec{a}$ is a unit vector, $\therefore |\vec{a}|=1$

As $(\vec{x}-\vec{a}).(\vec{x}+\vec{a})=12$

$\Rightarrow \vec{x}.\vec{x}+\vec{x}.\vec{a}-\vec{a}.\vec{x}-\vec{a}.\vec{a}=12$

$\Rightarrow |\vec{x}|^2-|\vec{a}|^2=12$.

$\Rightarrow |\vec{x}|^2-1^2=12$

$\Rightarrow |\vec{x}|^2=13$

$\therefore |\vec{x}|=\sqrt{13}$.

SECTION C

26. $\displaystyle\int\frac{dx}{\sqrt{3-2x-x^2}}$

$\displaystyle=\int\frac{dx}{\sqrt{-(x^2+2x-3)}}=\int\frac{dx}{\sqrt{4-(x+1)^2}}$

$\displaystyle=\int\frac{dx}{\sqrt{2^2-(x+1)^2}}$

$=\sin^{-1}\left(\dfrac{x+1}{2}\right)+C$.

27. P (not obtaining an odd person in a single round)

= P (all three of them throw tails or all three of them throw heads)

$=\dfrac{1}{2}\times\dfrac{1}{2}\times\dfrac{1}{2}\times2=\dfrac{1}{4}$

P (obtaining an odd person in a single round)

$=1-$ P (not obtaining an odd person in a single round) $=\dfrac{3}{4}$.

The required probability

= P ('in first round there is no odd person' and 'in second round there is no odd person' and 'in third round there is an odd person')

$=\dfrac{1}{4}\times\dfrac{1}{4}\times\dfrac{3}{4}=\dfrac{3}{64}$.

OR

Let X denote the Random Variable defined by the number of defective items.

$P(X=0)=\dfrac{4}{6}\times\dfrac{3}{5}=\dfrac{2}{5}$, $P(X=1)=2\times\left(\dfrac{2}{6}\times\dfrac{4}{5}\right)=\dfrac{8}{15}$, $P(X=2)=\dfrac{2}{6}\times\dfrac{1}{5}=\dfrac{1}{15}$.

The table for probability distribution is given below.

x_i	0	1	2
p_i	$\frac{6}{15}$	$\frac{8}{15}$	$\frac{1}{15}$

Mean $= \sum p_i x_i = 0 + \frac{8}{15} + \frac{2}{15} = \frac{10}{15} = \frac{2}{3}$.

28. Let $I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}} = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$ ...(i)

Using $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$, we get

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos\left(\frac{\pi}{6}+\frac{\pi}{3}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{6}+\frac{\pi}{6}-x\right)}+\sqrt{\cos\left(\frac{\pi}{6}+\frac{\pi}{3}-x\right)}}\,dx$$

$$\Rightarrow I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx \ldots\text{(ii)}$$

Adding (i) and (ii), we get

$$2I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx + \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$$

$$\Rightarrow 2I = \int_{\pi/6}^{\pi/3} dx$$

$$\Rightarrow 2I = [x]_{\pi/6}^{\pi/3} = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$$

Hence, $I = \frac{\pi}{12}$.

OR

$$\int_0^4 |x-1|\,dx = \int_0^1 |x-1|\,dx + \int_1^4 |x-1|\,dx$$

$$= \int_0^1 (1-x)dx + \int_1^4 (x-1)\,dx$$

$$= \left[x - \frac{x^2}{2}\right]_0^1 + \left[\frac{x^2}{2} - x\right]_1^4$$

$$= \left(1-\frac{1}{2}\right) - 0 + (8-4) - \left(\frac{1}{2}-1\right)$$

$$= 5.$$

29. $y\,dx + (x - y^2)dy = 0$

Reducing the given differential equation to the form $\frac{dx}{dy} + Px = Q$ we get, $\frac{dx}{dy} + \frac{x}{y} = y$

That means, $P = \frac{1}{y}$, $Q = y$.

$\therefore$ I.F. $= e^{\int P dy} = e^{\int \frac{1}{y} dy} = e^{\log y} = y$.

The general solution is given by $x\,(\text{I.F.}) = \int Q \times (\text{I.F.})\,dy$

$\Rightarrow xy = \int y^2 dy$

$\Rightarrow xy = \frac{y^3}{3} + C$, which is the required general solution.

OR

$x dy - y dx = \sqrt{x^2 + y^2}\,dx$

It is a Homogeneous Differential Equation as, $\frac{dy}{dx} = \frac{\sqrt{x^2 + y^2} + y}{x}$

$\Rightarrow \frac{dy}{dx} = \sqrt{\frac{x^2 + y^2}{x^2}} + \frac{y}{x}$

$\Rightarrow \frac{dy}{dx} = \sqrt{1 + \left(\frac{y}{x}\right)^2} + \frac{y}{x}$ i.e., $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$.

To solve, put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

So, $v + x\frac{dv}{dx} = \sqrt{1 + v^2} + v$

Separating variables, we get $\frac{dv}{\sqrt{1 + v^2}} = \frac{dx}{x}$

Integrating, we get $\log\left|v + \sqrt{1 + v^2}\right| = \log|x| + \log K$, $K > 0$

$\Rightarrow \log\left|y + \sqrt{x^2 + y^2}\right| = 2\log|x| + \log K$

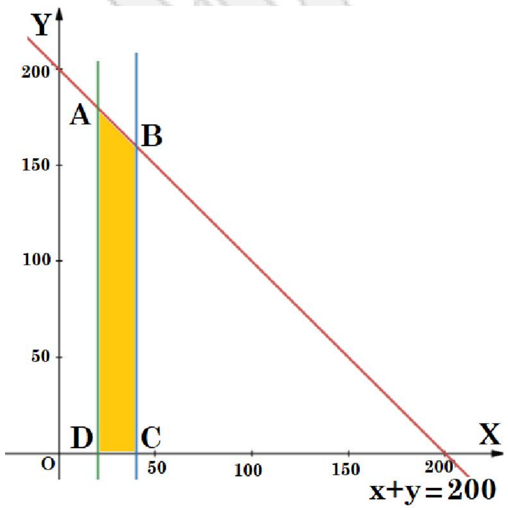
$\Rightarrow \log\left|y + \sqrt{x^2 + y^2}\right| = \log(x^2 K)$

$\Rightarrow y + \sqrt{x^2 + y^2} = \pm K x^2$

$\Rightarrow y + \sqrt{x^2 + y^2} = C x^2$, which is the required general solution.

30. Consider the graph shown here.

The corner points of the feasible region are $C(40, 0)$, $D(20, 0)$, $B(40, 160)$, $A(20, 180)$.



Corner point	Value of Z
$D(20, 0)$	8000
$C(40, 0)$	16000
$B(40, 160)$	64000 $\leftarrow$ Max.
$A(20, 180)$	62000

Maximum value of Z occurs at $x = 40$, $y = 160$.

And, $Z_{max} = 64000$.

31. $\int \frac{(x^3+x+1)}{(x^2-1)}dx = \int \left(x + \frac{2x+1}{(x-1)(x+1)} \right) dx$

Consider $\frac{2x+1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$

$\Rightarrow 2x+1 = A(x+1)+B(x-1)$

On comparing the coefficients of x and constant terms on both sides, we get

$A+B=2,\ A-B=1$

On solving these equations, we get $A=\frac{3}{2},\ B=\frac{1}{2}$.

Hence, $\int \frac{(x^3+x+1)}{(x^2-1)}dx = \int \left(x + \frac{3}{2(x-1)} + \frac{1}{2(x+1)} \right) dx$

$$= \frac{x^2}{2} + \frac{3}{2}\log|x-1| + \frac{1}{2}\log|x+1| + C$$

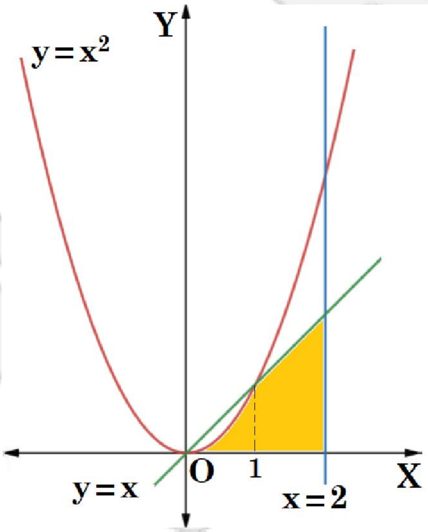
$$= \frac{x^2}{2} + \frac{1}{2}\log\left|(x-1)^3\right| + \frac{1}{2}\log|x+1| + C$$

$$= \frac{x^2}{2} + \frac{1}{2}\log\left|(x-1)^3(x+1)\right| + C.$$

SECTION D

32. Consider $y = x^2$ ...(i) and $y = x$ ...(ii)

On solving (i) and (ii), we get the points of intersection as $(0, 0)$ and $(1, 1)$.



Required Area $= \int_0^1 y_{parabola}dx + \int_1^2 y_{line}dx$

So, required area $= \int_0^1 x^2 dx + \int_1^2 x\,dx$

$\Rightarrow$ required area $= \left[\frac{x^3}{3}\right]_0^1 + \left[\frac{x^2}{2}\right]_1^2$

$\Rightarrow$ required area $= \frac{1}{3} + \frac{3}{2} = \frac{11}{6}$ Sq. units.

33. Let $(a, b) \in N \times N$. Then we have

$ab = ba$ (by commutative property of multiplication of natural numbers)

$\Rightarrow (a, b)R(a, b)$

Hence, R is reflexive.

Let $(a, b), (c, d) \in N \times N$ such that $(a, b)R(c, d)$. Then $ad = bc$

$\Rightarrow cb = da$ (by commutative property of multiplication of natural numbers)

$\Rightarrow (c, d)R(a, b)$

Hence, R is symmetric.

Let $(a, b), (c, d), (e, f) \in N \times N$ such that

$(a, b)R(c, d)$ and $(c, d)R(e, f)$.

Then $ad = bc$, $cf = de$

$\Rightarrow adcf = bcde$

$\Rightarrow af = be$

$\Rightarrow (a, b) R (e, f)$

Hence, R is transitive.

Since, R is reflexive, symmetric and transitive so, R is an equivalence relation on $N \times N$.

OR

Let $A \in P(X)$. Then $A \subset A$

$\Rightarrow (A, A) \in R$

Hence, R is reflexive.

Let $A, B, C \in P(X)$ such that

$(A, B), (B, C) \in R$

$\Rightarrow A \subset B, B \subset C$

$\Rightarrow A \subset C$

$\Rightarrow (A, C) \in R$

Hence, R is transitive.

Let $\phi, X \in P(X)$ such that $\phi \subset X$. Hence, $(\phi, X) \in R$.

But, $X \not\subset \phi$, which implies that $(X, \phi) \notin R$.

Thus, R is not symmetric.

34. The given lines are non-parallel lines. There is a unique line-segment PQ (P lying on one and Q on the other, which is at right angles to both the lines. PQ is the shortest distance between the lines. Hence, the shortest possible distance between the insects = PQ).

The position vector of P lying on the line $\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$ is $(6+\lambda)\hat{i} + (2-2\lambda)\hat{j} + (2+2\lambda)\hat{k}$ for some λ.

The position vector of Q lying on the line $\vec{r} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$ is $(-4+3\mu)\hat{i} + (-2\mu)\hat{j} + (-1-2\mu)\hat{k}$ for some μ.

Now $\overrightarrow{PQ} = (-10+3\mu-\lambda)\hat{i} + (-2\mu-2+2\lambda)\hat{j} + (-3-2\mu-2\lambda)\hat{k}$

Since, PQ is perpendicular to both the lines

$(-10+3\mu-\lambda)(1) + (-2\mu-2+2\lambda)(-2) + (-3-2\mu-2\lambda)(2) = 0$,

i.e., $\mu - 3\lambda = 4$ ...(i)

And $(-10+3\mu-\lambda)(3) + (-2\mu-2+2\lambda)(-2) + (-3-2\mu-2\lambda)(-2) = 0$,

i.e., $17\mu - 3\lambda = 20$ ...(ii)

Solving (i) and (ii) for λ and μ, we get $\mu = 1$, $\lambda = -1$.

The position vector of the points, at which they should be so that the distance between them is the shortest, are $5\hat{i} + 4\hat{j}$ and $-\hat{i} - 2\hat{j} - 3\hat{k}$

So, $\overrightarrow{PQ} = -6\hat{i} - 6\hat{j} - 3\hat{k}$

The shortest distance $= |\overrightarrow{PQ}| = \sqrt{6^2 + 6^2 + 3^2} = 9$ units.

OR

Eliminating t between the equations, we obtain the equation of the path $\frac{x}{2} = \frac{y}{-4} = \frac{z}{4}$, which is the equation of the line passing through the origin having direction ratios 2, –4, 4. This line is the path of the rocket.

When $t = 10$ seconds, $x = 2\times10 = 20$, $y = -4\times10 = -40$, $z = 4\times10 = 40$

So, the rocket will be at the point (20, –40, 40). Let P(20, –40, 40).

$\therefore$ The required distance from the origin at 10 seconds, $OP = \sqrt{20^2 + 40^2 + 40^2}$ km $= 60$ km.

Now, as the distance of a given point $P(\vec{p})$ from a Line $\vec{r} = \vec{a} + \lambda\vec{b}$ is $\dfrac{|(\vec{p} - \vec{a}) \times \vec{b}|}{|\vec{b}|}$.

Here $\vec{p} = 20\hat{i} - 40\hat{j} + 40\hat{k}$, $\vec{a} = 20\hat{i} - 10\hat{j} + 40\hat{k}$ $\quad\therefore (\vec{p} - \vec{a}) = -30\hat{j}$.

So, the distance of the point P(20, –40, 40) from $\vec{r} = 20\hat{i} - 10\hat{j} + 40\hat{k} + \lambda(10\hat{i} - 20\hat{j} + 10\hat{k})$ is

$$= \frac{|-30\hat{j} \times (10\hat{i} - 20\hat{j} + 10\hat{k})|}{|10\hat{i} - 20\hat{j} + 10\hat{k}|} \text{ km}$$

$$= \frac{|-300\hat{i} + 300\hat{k}|}{\sqrt{100 + 400 + 100}} \text{ km}$$

$$= \frac{300\sqrt{2}}{10\sqrt{6}} \text{ km} = 10\sqrt{3} \text{ km}.$$

35. For $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, $|A| = 2(0) + 3(-2) + 5(1) = -1$.

Also $\operatorname{adj} A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$

$$\therefore A^{-1} = \frac{\operatorname{adj} A}{|A|} = \frac{1}{(-1)}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

Given equations are $2x - 3y + 5z = 11$, $3x + 2y - 4z = -5$, $x + y - 2z = -3$

Let $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$.

$\because AX = B$

$\Rightarrow X = A^{-1}B$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 - 5 + 6 \\ -22 - 45 + 69 \\ -11 - 25 + 39 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\therefore x = 1,\ y = 2,\ z = 3$.

SECTION E

36. **(i)** $f(x) = -0.1x^2 + mx + 98.6$, being a polynomial function, is differentiable everywhere, hence, f is differentiable in $(0, 12)$.

(ii) $f'(x) = -0.2x + m$

Since, 6 is the critical point, $f'(6) = 0$

$\Rightarrow -0.2 \times 6 + m = 0$

$\Rightarrow m = 1.2$.

(iii) $f(x) = -0.1x^2 + 1.2x + 98.6$

$\Rightarrow f'(x) = -0.2x + 1.2 = -0.2(x-6)$

For $f'(x) = 0$, $-0.2(x-6) = 0 \ \Rightarrow x = 6$.

In the interval	Sign of $f'(x)$ is	Conclusion
(0, 6)	Positive	f is strictly increasing in (0, 6)
(6, 12)	Negative	f is strictly decreasing in (6, 12)

OR

(iii) $f(x) = -0.1x^2 + 1.2x + 98.6$

$\Rightarrow f'(x) = -0.2x + 1.2$ and $f''(x) = -0.2$.

For $f'(x) = 0$, $-0.2x + 1.2 = 0 \ \therefore x = 6$

As $f''(6) = -0.2 < 0$.

Hence, by second derivative test we can see that, $x = 6$ is a point of local maximum.

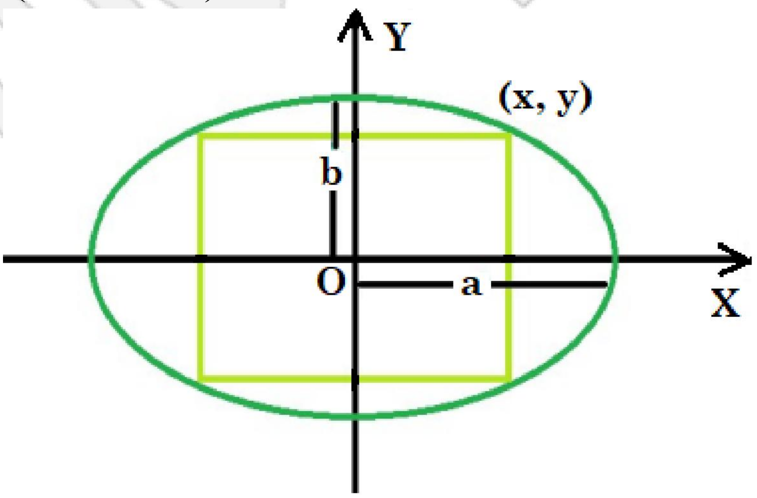
Also, the local maximum value $= f(6) = -0.1 \times 6^2 + 1.2 \times 6 + 98.6 = 102.2$.

We have $f(0) = 98.6,\ f(6) = 102.2,\ f(12) = 98.6$.

Now $x = 6$ is the point of absolute maximum and the absolute maximum value of the function $= 102.2$.

Also $x = 0, 12$ both are the points of absolute minimum and the absolute minimum value of the function $= 98.6$.

37. **(i)** Let $(x, y) = \left(x, \frac{b}{a}\sqrt{a^2 - x^2}\right)$ be the upper right vertex of the rectangle.



The area function, $A = 2x \times 2\frac{b}{a}\sqrt{a^2 - x^2}$

$\Rightarrow A = \frac{4b}{a}x\sqrt{a^2 - x^2}$, $x \in (0, a)$.

(ii) $\frac{dA}{dx} = \frac{4b}{a}\left[x \times \frac{-x}{\sqrt{a^2 - x^2}} + \sqrt{a^2 - x^2}\right]$

$\Rightarrow \frac{dA}{dx} = \frac{4b}{a} \times \frac{a^2 - 2x^2}{\sqrt{a^2 - x^2}} = -\frac{4b}{a} \times \frac{(\sqrt{2}x + a)(\sqrt{2}x - a)}{\sqrt{a^2 - x^2}}$

For $\frac{dA}{dx} = 0 \quad \Rightarrow x = \frac{a}{\sqrt{2}}$

So, $x = \frac{a}{\sqrt{2}}$ is the critical point.

(iii) For the values of x less than $\frac{a}{\sqrt{2}}$ and close to $\frac{a}{\sqrt{2}}$, $\frac{dA}{dx} > 0$ and for the values of x greater than $\frac{a}{\sqrt{2}}$ and close to $\frac{a}{\sqrt{2}}$, $\frac{dA}{dx} < 0$.

Hence, by the first derivative test, there is a local maximum at the critical point $x = \frac{a}{\sqrt{2}}$. Since there is only one critical point, therefore, the area of the soccer field is maximum at this critical point $x = \frac{a}{\sqrt{2}}$.

$\therefore$ For maximum area of the soccer field, its length should be $a\sqrt{2}$ units and its width should be $b\sqrt{2}$ units.

OR

(iii) $A = 2x \times 2\frac{b}{a}\sqrt{a^2 - x^2}$, $x \in (0, a)$.

Squaring both sides, we get $Z = A^2 = \frac{16b^2}{a^2}x^2(a^2 - x^2) = \frac{16b^2}{a^2}(x^2a^2 - x^4)$, $x \in (0, a)$.

($\because$ A is maximum when Z is maximum.)

Now $\frac{dZ}{dx} = \frac{16b^2}{a^2}(2xa^2 - 4x^3) = \frac{32b^2}{a^2}x(a + \sqrt{2}x)(a - \sqrt{2}x)$

For $\frac{dZ}{dx} = 0 \quad \Rightarrow x = \frac{a}{\sqrt{2}}$

Also, $\frac{d^2Z}{dx^2} = \frac{32b^2}{a^2}(a^2 - 6x^2)$

As $\left(\frac{d^2Z}{dx^2}\right)_{x=\frac{a}{\sqrt{2}}} = \frac{32b^2}{a^2}(a^2 - 3a^2) = -64b^2 < 0$.

So, by the second derivative test, there is a local maximum value of Z at the critical point $x = \frac{a}{\sqrt{2}}$.

Since there is only one critical point therefore, Z is maximum at $x = \frac{a}{\sqrt{2}}$.

Hence, A is maximum at $x = \frac{a}{\sqrt{2}}$.

$\therefore$ For maximum area of the soccer field, its length should be $a\sqrt{2}$ units and its width should be $b\sqrt{2}$ units.

38. **(i)** Let X be the event that the shell fired from A hits the plane and Y be the event that the shell fired from B hits the plane. The following four hypotheses are possible before the trial, with the guns operating independently:

$E_1 = XY$, $E_2 = \bar{X}\bar{Y}$, $E_3 = \bar{X}Y$, $E_4 = X\bar{Y}$.

Let E = the shell fired from exactly one of them hits the plane.

So, $P(E_1) = 0.3 \times 0.2 = 0.06$, $P(E_2) = 0.7 \times 0.8 = 0.56$, $P(E_3) = 0.7 \times 0.2 = 0.14$,

$P(E_4) = 0.3 \times 0.8 = 0.24$; $P(E|E_1) = 0$, $P(E|E_2) = 0$, $P(E|E_3) = 1$, $P(E|E_4) = 1$.

Now $P(E) = P(E_1).P(E|E_1) + P(E_2).P(E|E_2) + P(E_3).P(E|E_3) + P(E_4).P(E|E_4)$

$\therefore P(E) = 0.14 + 0.24 = 0.38$.

(ii) By Bayes' theorem,

$$P(E_3|E) = \frac{P(E_3).P(E|E_3)}{P(E_1).P(E|E_1) + P(E_2).P(E|E_2) + P(E_3).P(E|E_3) + P(E_4).P(E|E_4)}$$

$$\Rightarrow P(E_3|E) = \frac{0.14}{0.38} = \frac{7}{19}.$$

NOTE: The four hypotheses form the partition of the sample space and it can be seen that the sum of their probabilities is 1. The hypothesis E_1 and E_2 are actually eliminated as $P(E|E_1) = P(E|E_2) = 0$.

Alternative way of writing the solution:

(i) P (shell fired from exactly one of them hits the plane)

$$= P\begin{bmatrix}\text{(Shell from A hits the plane and Shell from B does not hit the plane)} \\ \text{or (Shell from A does not hit the plane and Shell from B hits the plane)}\end{bmatrix}$$

$= 0.3 \times 0.8 + 0.7 \times 0.2 = 0.38$.

(ii) P (Shell fired from B hits the plane | Exactly one of them hits the plane)

$$= \frac{P(\text{Shell fired from B hits the plane} \cap \text{Exactly one of them hits the plane})}{P(\text{Exactly one of them hits the plane})}$$

$$= \frac{P(\text{Shell from only B hits the plane})}{P(\text{Exactly one of them hits the plane})}$$

$$= \frac{0.14}{0.38} = \frac{7}{19}.$$

This sample paper has been issued by **CBSE** for **2022-23 Board Exams of class 12 Mathematics (041)**.
Note : We've **re-typed** the official sample paper and, also done the necessary corrections at some places. Apart from that, further illustrations have been added as well in some questions.
If you notice any error which could have gone un-noticed, please do inform us via **WhatsApp @ +919650350480 (message only)** or, via **Email** at **iMathematicia@gmail.com**

Let's learn **Math** with **smile**:-)

- O.P. GUPTA, Math Mentor
For video lectures, please visit YouTube.com/@theopgupta